ASSESS YOURSELF

1. Is 1.010010001 ...... an irrational number? If so, why?
2. Give examples of two irrational numbers, the product of which is
   (*i*) a rational number (*ii*) an irrational number
3. Simplify $(5+\sqrt{2})(3+\sqrt{5})$.
4. Simplify $\left(\sqrt[3]{x^2}\right)^{\frac{3}{2}}$.
5. Insert two rational numbers between $\frac{2}{3}$ and $\frac{5}{3}$.
6. Simplify $64^{-\frac{1}{3}} . 64^{\frac{1}{3}} - 343^{\frac{2}{3}}$.
7. Write two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$.
8. Find the value of x, if $\sqrt[3]{4x-7} = 5$.
9. Let x and y be rational and irrational numbers, respectively. Is $x + y$ necessarily an irrational number? Give an example in support of your answer.
10. Locate $\sqrt{3}$ on the number line. **[CBSE 2016]**
11. Express 2.5434343 .... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
12. Simplify $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$.
13. Evaluate $4 \times (81)^{-\frac{1}{2}} \times \left[81^{\frac{1}{2}} + 81^{\frac{3}{2}}\right]$.

1. Give an example of a cubic polynomial.

2. Find the degree of polynomial $4x^3 + 9x - 6x^5$.

3. Find the coefficient of x^3 in the expression

$p(x) = (3x - 4)(x^2 + 5x - 1)$.

4. Find the zeroes of $x^2 - 2x$.

5. Find the value of $346^2 - 345^2$.

6. Classify the following as linear, quadratic, cubic and constant polynomials:

   (i) $5x - 2\sqrt{2}$ (ii) $9 - 8u^2$

   (iii) $8 - x^3 + 2x^2$ (iv) 7

7. Find the zero of the polynomial

$p(x) = (x - 4)^2 - (x - 6)^2$.

8. By remainder theorem, find the remainder when $p(x) = 4x^3 - 3x^2 + 2x - 5$ is divided by $g(x) = 1 - 2x$.

9. If $y + 1$ is a factor of $ky^3 + y^2 - 2y + 4k - 10$, then find the value of k.

10. Factorise:

   (i) $42 - x - x^2$ (ii) $16x^2 + 25 - 40x$

41. If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

42. Prove that:

$$(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$$

[NCERT Exemplar]

11. Factorise: $2x^3 - x^2 - 13x - 6$.

12. Evaluate the following by using suitable identity:

(*i*) 101×99 (*ii*) 999^3

13. Without actually calculating the cubes, find the value of

(*i*) $(-14)^3 + 8^3 + 6^3$

(*ii*) $27^3 + (-14)^3 + (-13)^3$

14. If $3x - 2y = 13$ and $xy = 5$, find the value of $27x^3 - 8y^3$.

15. Simplify: $(p + q)^3 - (p - q)^3 - 6q(p^2 - q^2)$.

16. When $f(y) = y^4 - 4y^3 + 8y^2 - my + n$ is divided by $y + 1$ and $y - 1$, we get remainder as 10 and 16 respectively. Find the remainder if $f(y)$ is divided by $y - 3$.

17. If $x^2 - 1$ is a factor of $px^4 + qx^3 + rx^2 + sx + u$, show that $p + r + u = q + s = 0$.

18. If $x^2 + \frac{1}{x^2} = 66$, find the value of $x^3 - \frac{1}{x^3}$.

19. What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$. So that result is exactly divisible by $x^2 + x - 2$?

20. Using factor theorem, factorise the polynomial $x^4 - x^3 - 7x^2 + x + 6$.

Activity 1

OBJECTIVE

To construct a square-root spiral.

MATERIAL REQUIRED

Coloured threads, adhesive, drawing pins, nails, geometry box, sketch pens, marker, a piece of plywood.

METHOD OF CONSTRUCTION

1. Take a piece of plywood with dimensions 30 cm × 30 cm.
2. Taking 2 cm = 1 unit, draw a line segment AB of length one unit.
3. Construct a perpendicular BX at the line segment AB using set squares (or compasses).
4. From BX, cut off BC = 1 unit. Join AC.
5. Using blue coloured thread (of length equal to AC) and adhesive, fix the thread along AC.
6. With AC as base and using set squares (or compasses), draw CY perpendicular to AC.
7. From CY, cut-off CD = 1 unit and join AD.

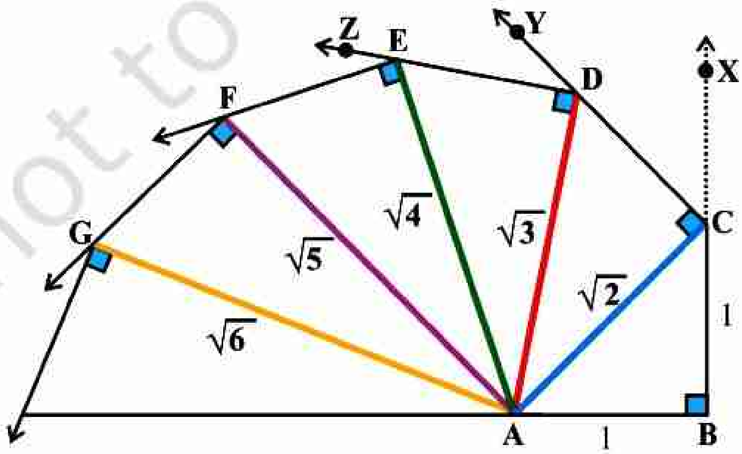

Fig. 1

8. Fix orange coloured thread (of length equal to AD) along AD with adhesive.
9. With AD as base and using set squares (or compasses), draw DZ perpendicular to AD.
10. From DZ, cut off DE = 1 unit and join AE.
11. Fix green coloured thread (of length equal to AE) along AE with adhesive [see Fig. 1].

    Repeat the above process for a sufficient number of times. This is called "a square root spiral".

Demonstration

1. From the figure, $AC^2 = AB^2 + BC^2 = 12 + 12 = 2$ or $AC = \sqrt{2}$.

   $AD^2 = AC^2 + CD^2 = 2 + 1 = 3$ or $AD = \sqrt{3}$.

2. Similarly, we get the other lengths AE, AF, AG, ... as $\sqrt{4}$ or 2, $\sqrt{5}$, $\sqrt{6}$ ....

Observation

On actual measurement

AC = ... , AD = ... , AE = ... , AF = ... , AG = ...

$\sqrt{2}$ = AC = ... (approx.),

$\sqrt{3}$ = AD = ... (approx.),

$\sqrt{4}$ = AE = ... (approx.),

$\sqrt{5}$ = AF = ... (approx.)

Application

Through this activity, existence of irrational numbers can be illustrated.

Activity 3

OBJECTIVE

To verify the algebraic identity :

$(a + b)^2 = a^2 + 2ab + b^2$

MATERIAL REQUIRED

Drawing sheet, cardboard, cello-tape, coloured papers, cutter and ruler.

METHOD OF CONSTRUCTION

1. Cut out a square of side length *a* units from a drawing sheet/cardboard and name it as square ABCD [see Fig. 1].
2. Cut out another square of length *b* units from a drawing sheet/cardboard and name it as square CHGF [see Fig. 2].

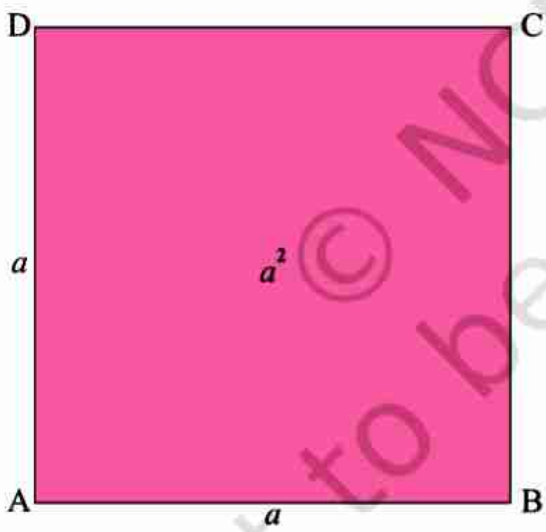

Fig. 1

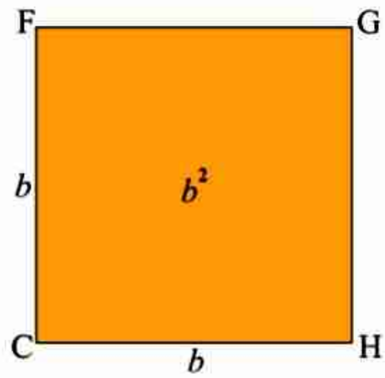

Fig. 2

3. Cut out a rectangle of length *a* units and breadth *b* units from a drawing sheet/cardbaord and name it as a rectangle DCFE [see Fig. 3].
4. Cut out another rectangle of length *b* units and breadth *a* units from a drawing sheet/cardboard and name it as a rectangle BIHC [see Fig. 4].

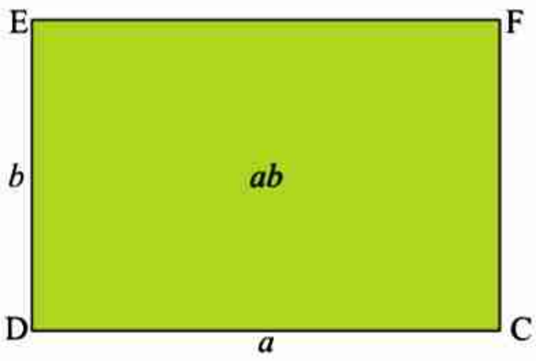

Fig. 3

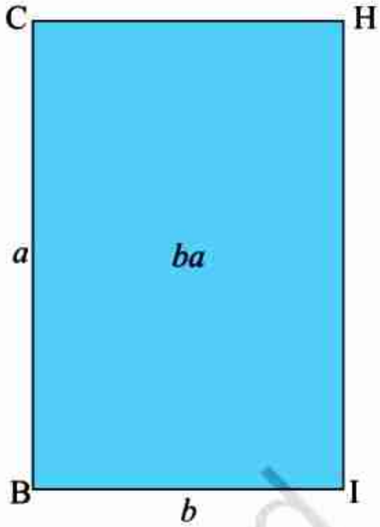

Fig. 4

5. Total area of these four cut-out figures

   = Area of square ABCD + Area of square CHGF + Area of rectangle DCFE + Area of rectangle BIHC

   $= a^2 + b^2 + ab + ba = a^2 + b^2 + 2ab.$

6. Join the four quadrilaterals using cello-tape as shown in Fig. 5.

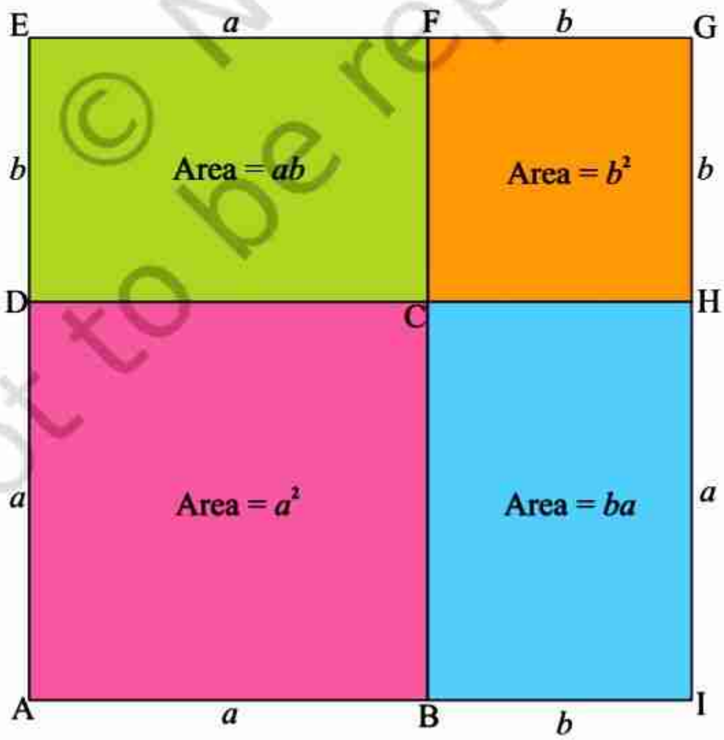

Fig. 5

Clearly, AIGE is a square of side $(a + b)$. Therefore, its area is $(a + b)^2$. The combined area of the constituent units $= a^2 + b^2 + ab + ab = a^2 + b^2 + 2ab$.

Hence, the algebraic identity $(a + b)^2 = a^2 + 2ab + b^2$

Here, area is in square units.

Observation

On actual measurement:

a = ..., b = ... $(a+b)$ = ...,

So, a^2 = ... b^2 = ..., ab = ...

$(a+b)^2$ = ..., $2ab$ = ...

Therefore, $(a+b)^2 = a^2 + 2ab + b^2$.

The identity may be verified by taking different values of a and b.

Application

The identity may be used for

1. calculating the square of a number expressed as the sum of two convenient numbers.
2. simplifications/factorisation of some algebraic expressions.

Activity 4

Objective

To verify the algebraic identity :

$(a - b)^2 = a^2 - 2ab + b^2$

Material Required

Drawing sheets, cardboard, coloured papers, scissors, ruler and adhesive.

Method of Construction

1. Cut out a square ABCD of side a units from a drawing sheet/cardboard [see Fig. 1].
2. Cut out a square EBHI of side b units ($b < a$) from a drawing sheet/cardboard [see Fig. 2].
3. Cut out a rectangle GDCJ of length a units and breadth b units from a drawing sheet/cardboard [see Fig. 3].
4. Cut out a rectangle IFJH of length a units and breadth b units from a drawing sheet/cardboard [see Fig. 4].

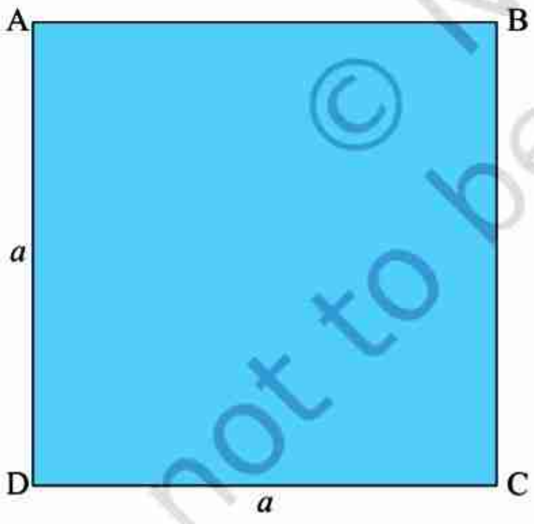

Fig. 1

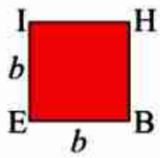

Fig. 2

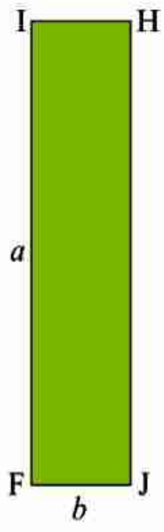

Fig. 4

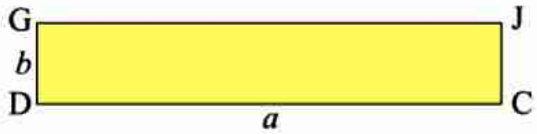

Fig. 3

5. Arrange these cut outs as shown in Fig. 5.

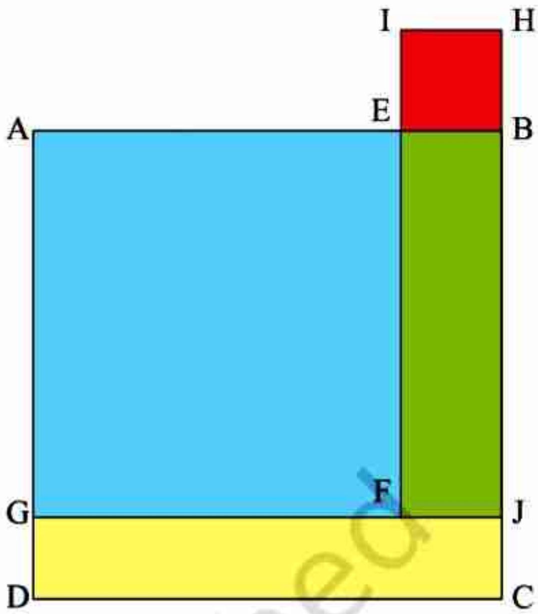

Fig. 5

Demonstration

According to figure 1, 2, 3, and 4, Area of square ABCD = a^2, Area of square EBHI = b^2

Area of rectangle GDCJ = ab, Area of rectangle IFJH = ab

From Fig. 5, area of square AGFE = AG × GF = $(a - b)(a - b) = (a - b)^2$

Now, area of square AGFE = Area of square ABCD + Area of square EBHI

– Area of rectangle IFJH – Area of rectangle GDCJ

$= a^2 + b^2 - ab - ab$

$= a^2 - 2ab + b^2$

Here, area is in square units.

Observation

On actual measurement:

a = ..., b = ..., $(a - b)$ = ...,

So, a^2 = ..., b^2 = ..., $(a - b)^2$ = ...,

ab = ..., $2ab$ = ...

Therefore, $(a - b)^2 = a^2 - 2ab + b^2$

Application

The identity may be used for

1. calculating the square of a number expressed as a difference of two convenient numbers.
2. simplifying/factorisation of some algebraic expressions.